Investment

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Overview

Aim for today: explore investment decision of a firm

- Marginal and average q

Technical aspects: optimization in infinite horizon problems

Investment decision of a firm:

- decreasing profits today
- accumulate bigger capital stock for tomorrow

dynamic setting $\Rightarrow$ infinite horizon
focus on capital accumulation $\Rightarrow$ ignore other factors of production

Basic notation and equations:

- Firms profits: $\pi(K_t) - I_t - \phi(I_t)$
- Capital accumulation $K_{t+1} = (1-\delta)K_t + I_t$

Firm's problem: maximize discounted profits:

$$\max_{\{I_\tau\}_{\tau=0}^{\infty}} \Pi = \sum_{\tau=0}^{\infty} \frac{1}{(1+r)^\tau} \left[\pi(K_\tau) - I_\tau - \phi(I_\tau)\right]$$

$$\text{s.t.} \quad K_{\tau+1} = (1-\delta)K_\tau + I_\tau$$

(one constraint for each period)

Solution method: form Lagrangian

$$\mathcal{L} = \sum_{\tau=0}^{\infty} \frac{1}{(1+r)^{\tau}} [\pi(K_{\tau}) - I_{\tau} - \phi(I_{\tau})] - \sum_{\tau=0}^{\infty} \lambda_{\tau} [K_{\tau+1} - (1-\delta)K_{\tau} - I_{\tau}]$$

λ_t: shadow value of the capital stock in period t
$\Rightarrow$ marginal impact of an exogenous increase in K_t on lifetime value of profits, valued from the perspective of the period t

Timing:

- λ_t shadow value at time t
- we want the present value - need to discount λ:

$$q_t = \frac{1}{(1+r)^t}\lambda_t$$

Then $\mathcal{L}$ can be written as

$$\mathcal{L} = \sum_{\tau=0}^{\infty} \frac{1}{(1+r)^\tau}\big(\pi(K_\tau) - I_\tau - \phi(I_\tau) - q_\tau[K_{\tau+1} - (1-\delta)K_\tau - I_\tau]\big)$$

FOC with respect to I_t:

$$\frac{\partial \mathcal{L}}{\partial I_t} = \frac{1}{(1+r)^t}(-1 - \phi'(I_t) + q_t)$$

Hence

$$1 + \phi'(I_t) = q_t$$

Interpretation: in equilibrium, the benefits of having capital (q_t) has to be equal to the costs.
Here the cost of making capital to work is equal to purchase costs (=1) and the installation costs (=$\phi'(I_t)$)

FOC with respect to K_{t+1}:

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \frac{1}{(1+r)^t}(-q_t) + \frac{1}{(1+r)^{t+1}}[\pi'(K_{t+1}) + q_{t+1}(1-\delta)] = 0$$

hence

$$\begin{aligned} \frac{1}{(1+r)^t} q_t &= \frac{1}{(1+r)^{t+1}}[\pi'(K_{t+1}) + q_{t+1}(1-\delta)] \\ (1+r)q_t &= \pi'(K_{t+1}) + q_{t+1}(1-\delta) \\ q_t - q_{t+1} + rq_t + \delta q_{t+1} &= \pi'(K_{t+1}) \end{aligned}$$

Hence $rq_t + \delta q_{t+1} = \pi'(K_{t+1}) + \Delta q_{t+1}$

$$rq_t + \delta q_{t+1} = \pi'(K_{t+1}) + \Delta q_{t+1}$$

- foregone interest gain
- cost of depreciation tomorrow
- Marginal revenue of K_{t+1}
- capital gains

interpretation: the returns to capital must be equal to the opportunity cost

$$q_t = \frac{1}{1+r}(\pi'(K_{t+1} + (1-\delta)q_{t+1}))$$

The value of capital today is equal to discounted value fo profits and the value fo capital tomorrow

Marginal q hard to measure $\Rightarrow$ average (Tobin's) q

$$\text{Tobin's } q = \frac{\text{total value of the firm}}{\text{replacement cost of the firms's capital}}$$

Summary

Investment:

- In investment, the firm balances the benefits with the opportunity costs of having more capital.
- Optimal decision is based on considering the costs/benefits of a marginal unit of capital marginal q
- marginal q difficult to measure $\Rightarrow$ Tobin's (average) q
- Technical aspects: optimization in infinite horizon problems