# Diploma Macro, Problem set 2, question 1 

Filip Rozsypal

March 4, 2012

Consider the Bernanke-Blinder extension to the ISLM model. Banks are assumed to hold bonds B, loans L and reserves R as assets, and have deposits D as liabilities, so that the representative banks balance sheet is:

$$
B+L+R=D
$$

Reserves are equal to the legal minimum reserve requirement $R=\tau D$, where $\tau=1 / 3$. This yields the supply of deposits in the money market:

$$
D^{s}=3 R
$$

The demand for deposits is given by the traditional money demand equation

$$
D^{d}=Y-\frac{1}{2} i_{B}
$$

where $Y$ is real aggregate output, $i_{B}$ the bond interest rate. The demand for loans is described by

$$
L^{d}=Y-\frac{1}{4}\left(i_{L}-i_{B}\right)
$$

where $i_{L}$ is the loan interest rate. The supply of loans is given by

$$
L^{s}=\frac{3}{4}(D-R)
$$

Goods market equilibrium is described by

$$
Y=60-\frac{1}{4}\left(i_{L}+i_{B}\right)
$$

(a) Derive the equilibrium bond interest rate $i_{B}$ in the money market in terms of output $Y$ and reserves $R$, and the equilibrium loan interest rate $i_{L}$ in the loan market in terms of $Y, R$ and $i_{B}$. Give an intuitive explanation.

- The interest rate $i_{b}$ is determined in the money market: $D^{s}=D^{d}$. Using this, we
get

$$
\begin{equation*}
i_{b}=2(Y-3 R) \tag{1}
\end{equation*}
$$

Interpretation: $D^{d}$ we can see the transaction demand for money, i.e. higher $Y$ generates higher demand. The supply is inelastic. This is the upward sloping LM curve. Note that demand for money is not affected by interest rate on loans, $i_{l}$. This is the case even in the Bernanke-Blinder paper.

- First note that $L^{s}=\frac{3}{2} R$, so the supply curve is a vertical line in $L \times i_{l}$ space. The demand is downward sloping (negative coefficient on $i_{l}$ ) and an increase in $Y$ and $i_{b}$ would shift the curve up $(Y)$ and down $\left(i_{b}\right)$. The first effect is caused by higher demand for credit with fixed amount of reserved (here completely exogenous and a monetary policy instrument). The second effect is caused by substitutability between credit and deposits.
The increase of $i_{l}$ hence can be caused by both, an increase in $Y$ or by an increase in $i_{b}$. $i_{l}$ is endogenous here, so it does not make sense to look what would be the effect of a change in $i_{l}$ on other variables, the causality goes in the opposite direction.
To get $i_{l}$, let's look at the equilibrium condition at the loans market $L^{d}=L^{s}$. Using this, we get

$$
\begin{equation*}
i_{l}=i_{b}+2(2 Y-3 R) \tag{2}
\end{equation*}
$$

hence now we have $i_{l}=i_{l}\left(i_{b}, Y, R\right)$.
(b) Derive output $Y$ in terms of $R$ and $i_{B}$ such that there is equilibrium in both the goods market and the loan market. Give an intuitive explanation.
Let's find CC curve, i.e. $Y=Y\left(R, i_{b}\right)$. Use the IS curve (so far in terms $Y=$ $Y\left(i_{l}, i_{b}\right)$ ) and plug in the result (2) (which is $i_{l}=i_{l}\left(i_{b}, Y, R\right)$ ) to obtain $Y=Y\left(i_{l}, i_{b}\right)=$ $Y\left(i_{l}\left(i_{b}, Y, R\right), i_{b}\right)=Y\left(i_{b}, R\right)$. The result (solved for $\left.i_{b}\right)$ is

$$
\begin{equation*}
i_{b}=(120+3 R)-4 Y \tag{3}
\end{equation*}
$$

The CC curve is downward sloping.
On more conceptual level, in this model, IS curve is given as a mapping $i_{b} \times y_{l} \rightarrow Y$. At the same time, LM curve is just mapping between $i_{b}$ and $Y$ (think of $R$ as an exogenous parameter). To relate IS to LM, we need somehow flatten the $i_{l}$ dimension of IS. This is possible, because we know what $i_{l}$ has to be in order to get an equilibrium on the loans market (given $Y$ and $i_{b}$ ), see equation (2).
(c) Suppose the central bank increases the level of reserves from $R=10$ to $R^{\prime}=12$. Compute the initial and new equilibrium level of output $Y$ and the bond interest rate $i_{B}$. Illustrate the effect graphically and provide an economic explanation.

The equilibrium is the point of intersection of LM and CC:

$$
\begin{align*}
2(Y-3 R) & =(120+3 R)-4 Y \\
\cdots &  \tag{4}\\
Y & =\frac{3}{2} R+20
\end{align*}
$$

(i) using $R=10$ we get $Y=35$ and $i_{b}=10$
(ii) using $R=12$ we get $Y=38$ and $i_{b}=4$

Hence the increase in the reserves increases output and decreases $i_{b}$.
(d) Suppose that the central bank subsequently raises the required reserve ratio to $\tau=1 / 2$ to address liquidity problems in the banking sector. Explain how this affects the equilibrium level of output $Y$ and the bond interest rate $i_{B}$. Provide a graphical illustration.

Now $\tau=1 / 2$. The LM curve can be written as

$$
i_{b}=2(Y-2 R)=\underbrace{2(Y-3 R)}_{\text {original } L M}+\underbrace{2 R}_{\text {shift }}
$$

so the effect of changing $\tau$ can be seen to shift the LM curve left/up.
Similarly, at the loans market,

$$
\begin{equation*}
i_{l}=i_{b}+2(2 Y-3 R)+3 R, \tag{5}
\end{equation*}
$$

so the CC curve takes form of

$$
\begin{equation*}
i_{b}=\underbrace{(120+3 R)-4 Y}_{\text {original CC }}-\underbrace{\frac{3}{2} R}_{\text {shift }}, \tag{6}
\end{equation*}
$$

so CC curve shifts left/down.
Observing new CC and LM curves, the slopes are not affected, but the both curves are shifted. The new equilibrium has lower output, but the effect on $i_{b}$ depends on the relative size of the shifts (possible to compute analytically).

