Week 2 period model

Filip Rozsypal

November 16, 2011

Outline

Two period model

Basic setting

Setting:

- agent lives for two periods, $t = 1, 2$
- in each period t she gets endowment ω_t
- interest rate exogenously given
- utility function: $U = u(c_1) + \beta u(c_2)$, with $u(\cdot)$ increasing and concave

Question: What is the optimal consumption profile (c_1, c_2)?
Why: this is the simplest possible setting with consumption/saving decision
Next step: try to expand out insights and discuss the role of govenrment

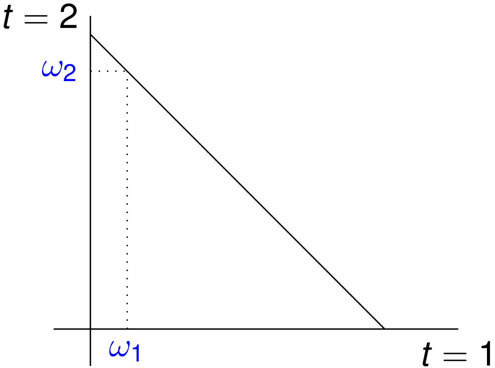
Budget constraint (BC):

$$c_1 + \frac{c_2}{1+r} = \omega_1 + \frac{\omega_2}{1+r}$$

- Interpretation: discounted life-time spending (on consumption) has to be (lower or) equal to discounted life-time income.
- Underlying assumption: no borrowing constraints

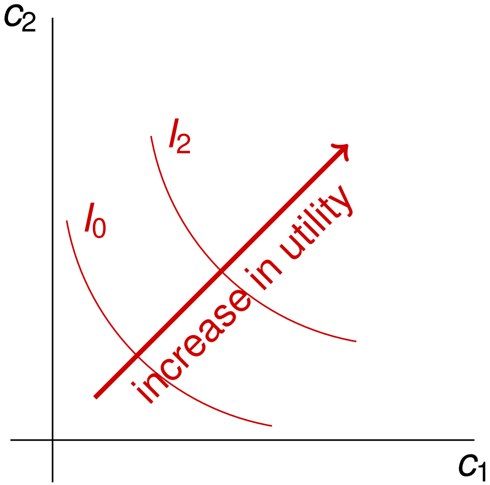
Easy to generalize to infinite horizon case:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{\omega_t}{(1+r)^t}$$



Budget constraint: line with $-(1 + r)$ slope intersecting (ω_1, ω_2)

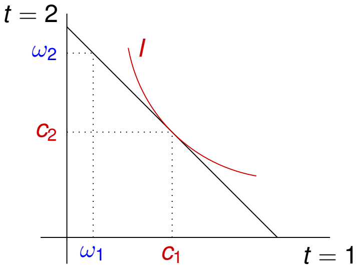
- If $(c_1, c_2) = (\omega_1, \omega_2)$, then no saving/borrowing is taking place,
- from this point, the consumer might consume more today (increasing c_1 by one unit) only by consuming less tomorrow (decreasing c_2 by $1 + r$)



Slope of indifference curves: $-\frac{u'(c_1)}{\beta u'(c_2)}$

Two period model

Optimal consumption profile



If (c_1, c_2) is optimal then the corresponding indifference curve has to be tangent to the budget constraint and
$\rightarrow$Euler equation is satisfied (see later)

Solution methods:

- Lagrange multipliers
- solve for one consumption and find FOC (with two periods only)

From BC:

$$c_2 = (1 + r)(\omega_1 - c_1) + \omega_2$$

Interpretation: consumption in the second (last) period is equal to the savings from the first period $(1 + r)(\omega_1 - c_1)$ (can be negative) combined with the income in the second period.
In other words, with ω_1 and ω_2 given, once c_1 is chosen, c_2 is also determined by the budget constraint.

Substituting this into the utility function allows us to represent a fucntion of two variables $U(c_1, c_2)$ as a function of one variable only $U(c_1)$

$$U(c_1, c_2) = u(c_1) + \beta u(c_2)$$
$$U(c_1) = u(c_1) + \beta u\big[(1+r)(\omega_1 - c_1) + \omega_2\big]$$

FOC:

$$U'(c_1) = u'(c_1) + \beta u'\big[(1+r)(\omega_1 - c_1) + \omega_2\big](-1)(1+r)$$
$$0 = u'(c_1) - (1+r)\beta u'\big[(1+r)(\omega_1 - c_1) + \omega_2\big]$$

substituting c_2 back and putting FOC equal to zero we get Euler Equation

$$u'(c_1) = (1+r)\beta u'(c_2)$$

Interpretation:

1. by postponing consumption of one marginal unit of consumption today I am decreasing my utility today by $u'(c_t)$
2. if I save this one unit of consumption, tomorrow I get $1 + r$, however, the utility is discounted by β
3. in equilibrium, no such a transfer is profitable $\Rightarrow$ marginal benefits must be equal

Euler Equation

$$u'(c_1) = (1 + r)\beta u'(c_2)$$

Observation: Neither ω_1 nor ω_2 are present in EE

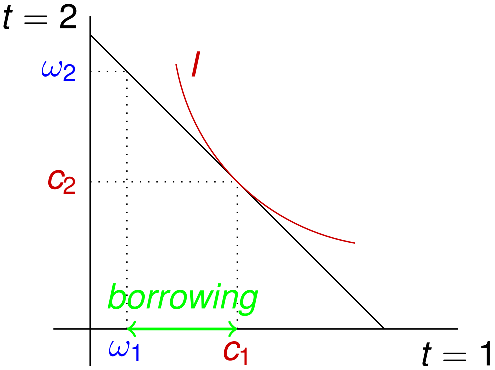
- consumption smoothing: time path of income is irrelevant for the path of consumption
  consumption profile is smoother then income profile
- no liquidity constraints is crucial assumption!

Euler Equation

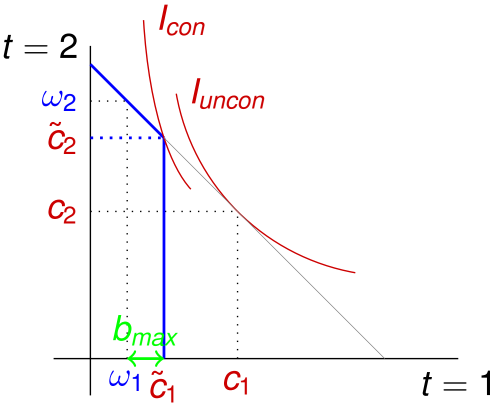
$$u'(c_1) = (1 + r)\beta u'(c_2)$$

Recall: the point of tangency of the indifference curve with th BC is characterised by $-(1 + r) = -\frac{u'(c_1)}{\beta u'(c_2)}$
This follows directly from Euler equation!

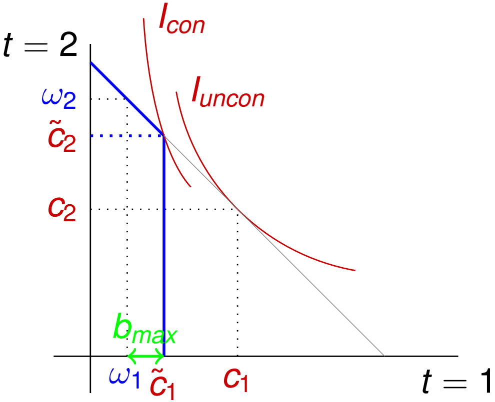
Note that if $c_1 > \omega_1$ then the consumer is a borrower in the first period



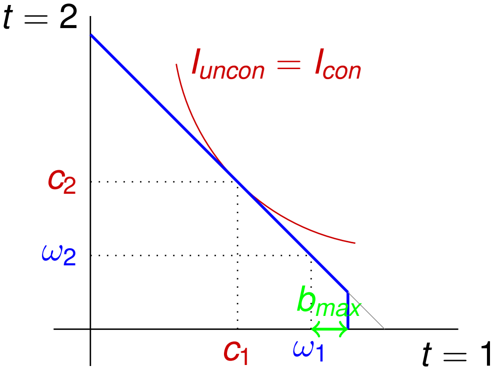
What if the consumers are liquidity constrained? (=there is some maximum borrowing b_{max})



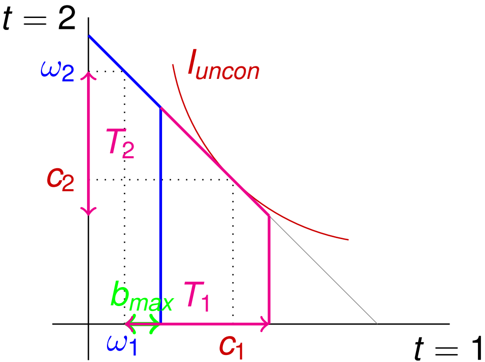
- Lower welfare attained: constrained indifference curve I_{con} is below the unconstrained one I_{uncon}



- Euler equation does not hold: I_{con} is not tangent to BC in the (constrained) optimal consumption profile $(\tilde{c}_1, \tilde{c}_2)$ ('corner solution')



Not always the case.
Other possible modification: higher r on borrowing then lending.
Exercise: how would the BC look like?



If the consumers are liquidity constrained, the government might help

- giving out transfers in the first period and taxing in the second period, so the government BC is satisfied
- positive transfer (=negative taxes) in the first period T_1 is paid by taxing T_2 in the second period

Outline

Ricardian Equivalence

Consider a situation without any borrowing constraint. Then transfers funded by taxes translate to movement along BC, hence the choice set is not affected. Can we generalize this?

- We have already established that the consumer does not care about time prife of income, he cares only about the discounted value of lifetime income $\sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}$
- Consequently, the consumer does not care about the taxes in individual periods, he cares only about the total discounted value of taxes $\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^t}$

Ricardian Equivalence

$\Rightarrow$ any change in the profile of government transfers, keeping $\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^t}$ constant, does not affect the optimal consumption profile (again: no borrowing constraints assumption critical)

- consumers simply change their borrowing profile
- considering only transfers, not government consumption (to keep interest rate unaffected)

Ricardian Equivalence

current policy debate

Ricardian equivalence $\Rightarrow$ any transfers cannot help to increase consumption

Some people use this to imply that the government cannot succeed in stimulating a low aggregate demand.

However,

- low income households are liquidity constrained
- behavioral economics demonstrates the limits of rationality (hence difficult to maintain assumptions about complete rationality, infinite horizon etc.)

Optional reading: Greg Mankiw, The Savers-Spenders Theory of Fiscal Policy, 2000 American Economic Review.

Neoclassical vs Keynesian consumers

Mankiw argues that significant number of households behave as rule of thumb consumers

- do not rationalize that extra transfers will eventually have to be paid back by taxes in the future
- behavior described better by some simple rule (i.e. spend 2/3 of your monthly income) rather then as a result of an optimization over infinite horizon

Such consumers can be describe by $C = a + bY$, i.e. the traditional keynesian consumption function.
Interesting area of research,

- this heterogeneity effects the evaluation of economic policies
- possible applications: life-cycle models and evaluation of pension reforms

Outline

Summary

- Two period model
  - setting
  - solution
  - assumptions and their representation
- Ricardian equivalence
  - consumption invariant to changes in timing of taxes and transfers (and corresponding assumptions)
  - Keynesian and rule of the thumb consumers