Problem set 3

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Question 1 (Romer, 8.1)

Consider a firm that produces output using a Cobb-Douglas combination of capital and labour $Y = K^{\alpha}L^{1-\alpha}$, $1 < \alpha < 1$. Suppose that the firm's price is fixed in the short run; thus it takes both the price of its product, P, and the quantity, Y as given. Input markets are competitive; thus the firm takes the wage, W and the rental price of capital, r, as given.

(i) What is the firm's choice of L given P, Y, W and K?

(ii) Given this choice of L, what are the profits as a function of P, Y, W and K?

(iii) Find the first order condition for the profit-maximizing choice of K. Is the second order condition satisfied?

(iv) Solve the first order condition from (iii) for K as a function of P, Y, W and r. How, if at all, do changes in each of these variables affect K?

Question 2 (Dr. Rendahl, Supervision 2, Problem 2)

Consider the following two-period investment problem. A firm has production function $Y = F(K)$ where F is concave ($F' > 0$ and $F'' < 0$). In period 1 it decides how much to invest, I_1, in order to produce output in period 2. The capital stock of the firm in period 2, K_2, is given by $K = (1 - \delta)K_1 + I_1$, where δ is the depreciation rate, $0 < \delta < 1$, and K_1 is the pre-determined level of capital stock. Assume that one unit of capital costs the same as one unit of output and normalize this price to 1. Suppose the firm maximizes the present discounted value of profits, Π, which are defined as

$$\Pi = F(K_1) - I_1 + \frac{F(K_2)}{1+r} + \frac{(1-\delta)K_2}{1+r}$$

(i) Derive the optimal demand for capital and give an economic interpretation of the condition. where r is the real interest rate.

(ii) Suppose now that firms have to pay an additional installation cost $\phi(I_1/K_1)$ when investing I_1. Assume that installation costs are convex ($\phi' > 0, \phi'' < 0$). Profots are therefore given by

$$\Pi = F(K_1) - I_1\left[1 + \phi\left(\frac{I_1}{K_1}\right)\right] + \frac{F(K_2)}{1+r} + \frac{(1-\delta)K_2}{1+r}$$

Derive the optimal demand for capital as a function of the marginal adjustment cost q defined as

$$q = 1 + \phi\left(\frac{I_1}{K_1}\right) + \left(\frac{I_1}{K_1}\right) \times \phi'\left(\frac{I_1}{K_1}\right)$$

(iii) Provide an economic interpretation of q. In particular, relate the answer from part (i) to the answer from part (ii) and explain the role of q in determining investment.