Problem set 2

Filip Rozsypal

November 23, 2011

Exercise 1

In problem set 1, Question 4, you were supposed to contrast effects of *permanent* and *temporary* changes in government consumption G. Does Ricardian equivalence hold in this setting?

Exercise 2 (Williamson, Chapter 8, Problem 5)

Suppose that the government introduces a tax on interest earnings. That is, borrowers face a real interest rate before and after the tax is introduced, but the lenders receive an interest rate of $(1-x)r$ on their savings, where x is the tax rate.

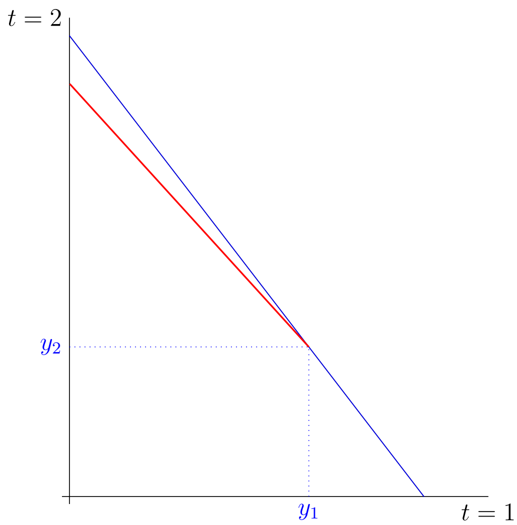
(i) Show the effects of the increase in the tax rate on a consumer's two period budget constraint.

(ii) How does the increase in the tax rate affect the optimal choice of consumption and saving for the consumer? Show how income and substitution effects matter for your answer, and show how it matters whether the consumer is initially a borrower or a lender.

Solution

(i) Define savings in the first period as $s_1 = y_1 - c_1$. Then

$$c_2 = \begin{cases} (1+r)(y_1-c_1)+y_2 & \text{if } s_1 \leq 0, \\ (1+(1-x)r)(y_1-c_1)+y_2 & \text{if } s_1 > 0 \end{cases}$$



What is the effect on the budget constraint? As long as the consumer is not lending in the first period, there is no effect, so the points where $c_1 \geq y_1$ are not affected (blue line).

If the consumer is lending ($c_1 < y_1$), then the savings are taxed, so effectively, effectively she is facing a lower *real* interest rate. This pivots teh budget constraint to the left along the point (y_1, y_2) (if $(c_1, c_2) = (y_1, y_2)$ then the consumer is neither lender nor saver and hence this corresponds to the 'highest' savings where she is not affected by the taxation).

(ii) There are 2 possible cases; the consumer can be either borrower or lender before the change in the taxation.

- Assume that the consumer was a borrower before the change in the taxes. Then the set feasible consumptions is smaller (lending is less profitable), however, the allocation which she chose before the change in the taxes is still feasible, hence it still must be the best allocation in the smaller feasible set. Hence the consumer chooses the same allocation and she remains a borrower in the first period. Finally, as she chooses the same allocation, her utility is not affected.
- On the other hand, if the consumer was originally a lender, then the original allocation is no longer feasible, hence she must be worse off.

  The substitution effect will induce the consumer to consume more today, ase saving has became less profitable. However, the income effect is negative and hence both c_1 and c_2 are reduced.

  The resulting c_2 will be lower after the change in the taxes (both income and substitution effect are negative). The change in c_1 is ambiguous as it depends on the relative magnitude of the income and substitution effect, which in turn depend on the particular utility fucntion.

Exercise 3 (Williamson, Chapter 8, Problem 8)

Consider a two period model of consumption. Assume a consumer who has current-period income $y_1 = 200$, future period income $y_2 = 150$, current and future taxes $T_1 = 40, T_2 = 50$, respectively, and faces a market real interest rate of $r = 5\%$. The consumer would like to consume equal amounts in both periods; that is, she would like to to set $c_1 = c_2$, if possible. However, the consume ia faced with a credit market imperfection, so she cannot borrow at all.

(i) Show the consumer's lifetime budget constraint and the indifference curves in a diagram.

(ii) Calculate her optimal current-period and future period consumption and optimal saving, and show this in the diagram.

(iii) Suppose that everything remains unchanged, except now $T_1 = 20$ and $T_2 = 71$. Calculate the effects on current and future consumption and optimal saving and show this in the diagram.

(iv) Suppose alternatively that $y_1 = 100$. Repeat parts (i)-(iii) and explain any differences

Solution

Preliminiaries

- Let's define the lifetime wealth as

$$W = y_1 - T_1 + \frac{y_2 - T_2}{1 + r}$$

- let's focus on saving in the period 1, s_1:

$$s_1 = y_1 - T_1 - c_1$$

then c_2 can be written as

$$c_2 = (1+r)s_1 + y_2 - T_2$$

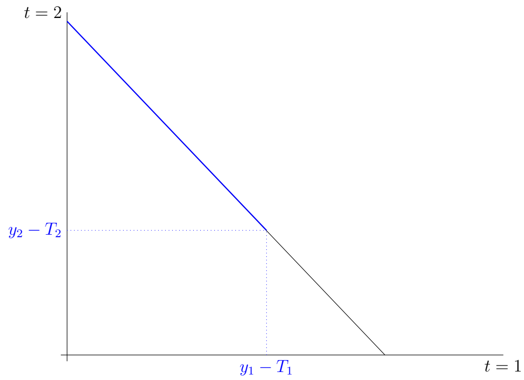
The Utility $U(c_1, c_2)$ can be rewritten as $U(s_1)$:

$$\begin{aligned} U(c_1, c_2) &= u(c_1) + \beta u(c_2) \\ &= u(y_1 - T_1 - s_1) + \beta u((1+r)s_1 + y_2 - T_2) \equiv U(s_1) \end{aligned}$$

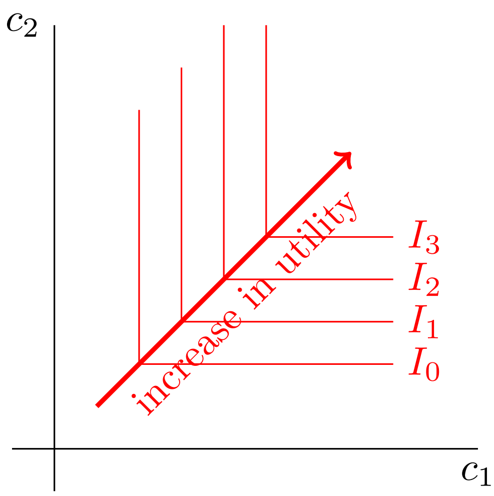
and hence 2 variable problem can be reduced to univariate problem.

Solution

(i) Budget constraint:



Indifference curves, I assume that $U(c_1, c_2) = min(c_1, c_2)$ (Leontief Utility Function), hence indifference curves look like this:



Such utility function will simplify a lot the algebra.

(ii) (a) Assume interior solution (=constraint not binding) Given the Leontief preferences, the (unconstrained) optimal allocation satisfies $c_1 = c_2 \equiv c$. Substituting this into the budged constraint gives

$$
\begin{aligned}
c + \frac{c}{1+r} &= y_1 - T_1 + \frac{y_2 - T_2}{1+r} = W \\
c(1 + \frac{1}{1+r}) &= W \\
c &= \frac{1+r}{2+r} W
\end{aligned}
$$

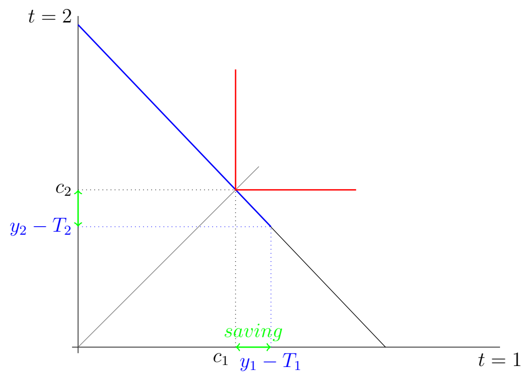
(b) check that the optimal allocation satisfied the constraint
I this case that means that $s_1 \geq 0$, as negative savings means borrowing.

$$
\begin{aligned}
s_1 &= y_1 - T_1 - c_1 \\
&= y_1 - T_1 - \frac{1+r}{2+r} W \\
&= y_1 - T_1 - \frac{1+r}{2+r}\left(y_1 - T_1 + \frac{y_2 - T_2}{1+r}\right) \\
&= \frac{1}{2+r}\Big((y_1 - T_1) - (y_2 - T_2)\Big)
\end{aligned}
$$

interpretation:

- saving is smaller the smaller is the difference between the disponible income at $t = 1$ and $t = 2$ (=consumption smoothing)
- due to the Leontief utility function, the consumers are induced to change their consumption only by the wealth effect, the substitution effect is switched off
- borrowing constraint is satisfied iff disponible income today is higher then tomorrow, $y_1 - T_1 \geq y_2 - T_2$

Because $S_1 > 0$, the constraint it not binding and the found we have found by FOC is the utility maximizer. The optimal allocation can be seen here:



(iii) Now we have $\tilde{T}_1 = 20$ and $\tilde{T}_2 = 71$. Comparing the disponible incomes we get

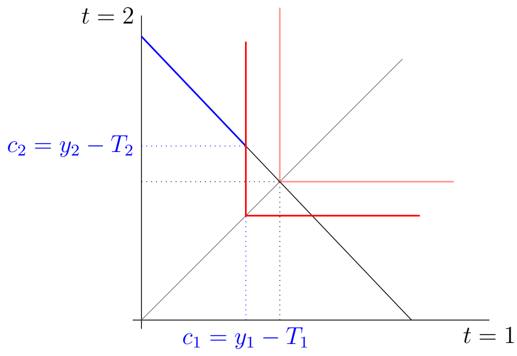
$$
y_1 - \tilde{T}_1 > y_2 - \tilde{T}_2
$$

hence by the same argument the constraint is not binding and the optimal consumption is $c = \frac{1+r}{2+r}\tilde{W}$, where $\tilde{W} = y_1 - \tilde{T}_1 + \frac{y_2 - \tilde{T}_2}{1+r}$.

(iv) Now consider situation where $\tilde{y}_1 = 100$ and $\tilde{y}_2 = 150$. By comparing the disponible incomes, we find that

$$
\tilde{y}_1 - T_1 < \tilde{y}_2 - T_2
$$

and hence the optimal point is no longer attainable, because having $c_1 = c_2$ would require the consumer to borrow in the first period.



Now, why we do have corner solution? We consider the following argument:

(a) The utility function is continuous and the feasible set is compact, hence the image of the utility function on the feasible set is bounded and there is a maximum and a maximizer.

(b) We computed the unconstrained optimum, but at this point the constraint is not satisfied, so this point is infeasible.

(c) No point in the interior of the set where the constraint is satisfied cannot be optimal, because it was, it would have to be the solution to the FOC. But the FOC had only one solution that that was out of this set.

(d) Hence because we have just ruled out any interior point of the feasible set, and we know that there is a maximum, therefore the maximum must be attained at the boundary of the feasible set as it is the only remaining point which we have not ruled out.